Scaling of Buffer and Capacity Requirements of Voice Traffic in Packet Networks

Florin Ciucu Oliver Hohlfeld
Deutsche Telekom Labs / Technische Universität Berlin
{florin, oliver}@net.t-labs.tu-berlin.de

***Abstract*—The migration of voice communication from the Public Switched Telephone Network to the Internet pushes the need to adequately size network resources such as buffers and capacity. This paper addresses the problem of how these resources should be scaled in the number of voice flows N in order to guarantee predefined packet loss probabilities and end-to-end delays. By deriving non-asymptotic buffer overflow probabilities at both edge and interior network nodes, the paper demonstrates that $O(1)$ buffers are sufficient to ensure probabilistic packet loss constraints at all utilizations. Also, by deriving end-to-end delay bounds, the paper shows that the required per-flow capacities are bounded by $O\left(\frac{1}{N}\right)$ when probabilistic end-to-end delay guarantees are sought. Numerical examples illustrate that statistical multiplexing dominates the effect of scheduling in multi-nodes scenarios with high capacities.**

I. Introduction

The problem of network dimensioning concerns with the allocation of network resources such that the transmitted data meets some Quality of Service (QoS) requirements. For instance, in the Public Switched Telephone Network (PSTN), the capacity of a telephone switch can be dimensioned using the classical Poisson arrival model for telephone calls [19] and the Erlang's loss formula [20], in order to meet call drop probabilities typically below one percent. These early works of Erlang on PSTN modelling and analysis are broadly regarded as representing the foundation for the classical queueing theory.

Since much recently, voice communication started to migrate from the PSTN to the Internet. Because PSTN and Internet's underlying technologies are fundamentally different, i.e., circuit-switched vs. packet-switched, network dimensioning solutions for the PSTN are not directly applicable to the Internet. In fact, the analysis of a packet-switched network is a much harder problem due to the interleaving of packets from multiple flows on the same link (statistical multiplexing) and the presence of buffers at the packet switches. This paper provides analytical results on loss and end-to-end delays, which can be in principle used to predict the Quality of Experience (QoE) level of voice calls with the ITU's E-Model (see Eq. (1) in [24]).

A voice source can be abstracted as a sequence of alternating active and silence periods; during the active period the source produces data at some constant rate and during the silence period the source is idle. The consideration of both active and silence periods, rather than just of the active period, leads to significant resource savings in a network carrying data produced by multiple voice flows. For instance, the application of digital speech interpolation in time division multiple access (TDMA) satellite systems can yield capacity savings at the Earth stations of as large as 50% [22]. Similar savings can also be reached in packet networks due to the underlying statistical multiplexing principle.

Since the active and silence periods of a voice source are reasonably well represented by exponential random variables, a commonly used analytical model for voice is the Markov-modulated On-Off (MMOO) process [31]. An exact derivation of buffer overflow probabilities when multiplexing many MMOO processes has been carried out for a single server with infinite sized buffer in the seminal paper of Anick, Mitra, and Sondhi [1]. The analysis is based on a fluid representation of the arrival MMOO processes and a constant rate server, meaning that the traffic and service units are infinitesimal. The buffer overflow probability is expressed as a sum of negative exponentials where the exponents are computable eigenvalues. For numerical purposes, the sum is usually approximated by the term containing the smallest negative eigenvalue, which dominates the sum; this dominant term is independent of the number of flows [31]. For other traffic models for voice multiplexers we refer to [17].

In this paper we analyze the fluid MMOO model from [1] in a scenario with many nodes by adopting a network calculus approach. With this approach, the arrival processes are represented using bounds on their moment generating functions (MGF), and the service received by a subset of arrivals at a node is represented in terms of probabilistic lower bounds set by service curves [9], [23]. While the obtained results in the single node case are provided only in terms of bounds (e.g. on the buffer overflow probability), the key advantage of the network calculus approach is that it can be easily extended to multi-node scenarios. The idea behind the simplification of analyzing multi-node scenarios is that it is generally much easier to provide bounds on the nodes' departures than to exactly characterize them. Using network calculus techniques we are able to derive buffer overflow probabilities at interior network nodes and also end-to-end delay bounds. These results are then directly applied to two network problems: (1) buffer dimensioning in order to guarantee certain loss probabilities, and (2) the minimum per-flow capacity at the nodes in order to guarantee probabilistic end-to-end delays.

We find that $O(1)$ buffers, in the number of flows N, are sufficient to guarantee any predefined loss probabilities at all utilizations in a network with FIFO scheduling carrying voice traffic. The $O(1)$ buffering scheme generally applies to traffic with exponentially decaying tails (e.g. voice) carried over open-loop UDP. This parallels a known buffering scheme for TCP traffic which argues that $O(1)$ buffers suffice in order to guarantee high link utilizations [18]. The result from [18] improved earlier buffer dimensioning schemes for TCP flows which suggested $O(N)$ (the bandwidth-delay product formula [33]) and $O(\sqrt{N})$ buffer sizing [2].

Also, we find that $O\left(\frac{1}{N}\right)$ per-flow capacities suffice in order to guarantee probabilistic end-to-end delays in a network with general scheduling. This is a direct consequence of the derived end-to-end delays which decay exponentially fast in the number of flows N. Moreover, numerical examples illustrate that by increasing N, or equivalently the capacity, most of the statistical multiplexing gain is captured, and also scheduling is dominated by statistical multiplexing.

From a technical point of view, our contribution is an extension of the network calculus formulation from [9], [23] to a continuous time setting appropriate to analyze the MMOO model from [1]. Also, we extend a service curve available for FIFO scheduling [16] in a probabilistic framework. In this way, we are able to compute *non-asymptotic* backlog bounds at interior network nodes and show that they behave as in a decomposed network where the upstream nodes can be ignored [21]. Similar decomposition results have been reported recently from an asymptotic point of view [32], [35], [21] and non-asymptotically but under additional independence assumptions of the arrivals (e.g. statistically independent increments) [13]. Our results further advocate the use of FIFO–perhaps the simplest algorithm to minimize the amount of time and hardware for resolving contention among arrival flows–by providing evidence that it can significantly simplify network analysis using decomposition arguments.

The network calculus approach to analyze queueing systems in terms of bounds, instead of exact results, was proposed by Cruz [15]. Initially formulated as an analytical tool to compute deterministic backlog and delay bounds (i.e. never violated), the network calculus was later extended in a probabilistic framework in order to exploit statistical multiplexing [37], [8], [26], [4]. When carrying out a probabilistic analysis, a key step commonly used in the calculus to compute backlog or delay bounds is to use the Chernoff bound together with the inequality

$$Pr\left(\sup_s X(s) > \sigma\right) \leq \sum_s Pr\left(X(s) > \sigma\right) \ , \qquad (1)$$

for some random process $X(s)$ with negative mean and $\sigma \geq 0$. Sharper bounds for the left-hand side term of Eq. (1) can be obtained for Markovian traffic using supermartingales [7], [10] or matrix analytical techniques [29].

The Inequality (1) can yield tight bounds for Poisson arrivals [12], due to their additivity. On the other hand, it can yield overly conservative bounds for arrivals burstier than Poisson [11], which is the case of MMOO flows. This is due to the fact that the combination of the Chernoff bound and the Inequality 1 is closely related to the effective bandwidth approximation for the steady-state backlog B

$$Pr\left(B > \sigma\right) \approx e^{-a\sigma} \ ,$$

which is invariant to the number of traffic flows N (here, a is a constant). This approximation was corrected to

$$Pr\left(B > \sigma\right) \approx e^{-NI\left(\frac{\sigma}{N}\right)} \ ,$$

where $I(\cdot)$ is a shape function depending on the cumulant generating function of the arrivals [5]. This improvement suggests that the buffer overflow probabilities decay exponentially in N (see also [11] for numerical evidence). Similar scalings have been obtained using the Bahadur-Rao inequality [3] in [30], [28]. However, these results, as well as those from [7], [10], [29], do not readily extend to interior network nodes, for which reason we will resort in our analysis on the combination of Chernoff bound and Inequality 1.

The rest of the paper is structured as follows. In Section II we review MMOO processes and introduce the main network calculus models for arrivals and service. In Section III we address the problem of buffer dimensioning and provide backlog bounds in a network with two nodes. In Section IV we address the problem of capacity dimensioning in a network such that the end-to-end delays are below a certain threshold. Brief conclusions are finally presented in Section V.

II. Arrival and Service Models

We use a continuous time model. The arrivals and departures at a node are modelled with non-decreasing, left-continuous processes. Each node serves the arrivals in a fluid-flow manner and stores the backlog in an infinite sized buffer. For any arrival process $A(t)$ we introduce for convenience the bivariate process $A(s,t) = A(t) - A(s)$. We also assume the initial condition $A(0) = 0$ and the causal condition $D(t) \leq A(t)$, where $D(t)$ is the departure process. The backlog process is denoted by $B(t) = A(t) - D(t)$, and the delay process is denoted by $W(t) = \inf\left\{d : A(t-d) \leq D(t)\right\}$.

To model voice traffic we use Markov-modulated processes [31]. Such a process is based on a homogenous and continuous-time Markov chain $X(t)$ with two states denoted by 'On' and 'Off', and with the transition matrix

$$Q = \begin{pmatrix} -\mu & \mu \\ \lambda & -\lambda \end{pmatrix} .$$

Here, μ and λ represent the transition rates from the 'On' state to the 'Off' state, and vice-versa, respectively. In the steady-state, the average dwell time of the process $X(t)$ in the 'On' state is $\frac{1}{\mu}$, and the average dwell time in the 'Off' state is $\frac{1}{\lambda}$.

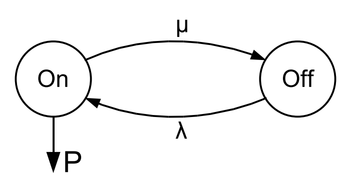

Fig. 1: A Markov-modulated On-Off traffic voice model.

Then, a continuous-time arrival process $A(t)$ is a Markov-modulated On-Off process driven by the Markov process $X(t)$ if its instantaneous arrival rate is either P or zero, depending whether $X(t)$ is in the 'On' or 'Off' states, respectively (see Figure 1).

Average 'On' time	Average 'Off' time	Peak rate	Average rate
$\frac{1}{\mu} = 0.4\ s$	$\frac{1}{\lambda} = 0.6\ s$	$P = 64\ Kb/s$	$r = 25.6\ Kb/s$

TABLE I: Parameters for a voice source.

For numerical illustrations we choose the values from Table I for the parameters of a voice flow. These values suggest that as much as $P/r = 2.5$ more voice flows can be potentially admitted using packet-switched schemes than when using pure circuit-switched schemes [31].

We now briefly introduce the network calculus models to represent arrivals and service. Arrivals are modelled using bounds on their MGFs [9], [23]. Concretely, we assume that the MGF of an arrival process $A(t)$ is bounded in the sense that for all $\theta > 0$ there exists a rate $r > 0$, which depends on θ, such that for all $t \geq 0$

$$\sup_{s \geq 0} E\left[e^{\theta A(s,s+t)}\right] \leq e^{\theta r t} \ . \tag{2}$$

When this holds we say that $A(t)$ is bounded by an MGF envelope with rate r; this rate is usually referred in the literature as the effective bandwidth of $A(t)$ [25]. In particular, for the MMOO process from Figure 1, the rate r satisfies the inequality [14]

$$r \leq \frac{1}{2\theta}\left(P\theta - \mu - \lambda + \sqrt{(P\theta - \mu + \lambda)^2 + 4\lambda\mu}\right) \ .$$

An important property of MGF envelopes is that if a number N of statistically independent flows $A_i(t)$ have MGF envelopes with rates r_i for some $\theta > 0$, respectively, then the aggregate flow $A(t) = \sum_{i=1}^{N} A_i(t)$ has an MGF envelope with the additive rate $r = \sum_{i=1}^{N} r_i$. This additive property is instrumental to exploit the statistical multiplexing at a node with multiple inputs.

While the arrivals are described using upper bounds on their moment generating functions, the service given by a node to a flow, or an aggregate of flows, is described by probabilistic lower bounds set by service curves. Concretely, a random process $\mathcal{S}(s,t)$ is a statistical service curve (referred in [9] as a dynamic F-server) for the arrivals $A(t)$ if the departures $D(t)$ satisfy for all $t \geq 0$

$$D(t) \geq A * \mathcal{S}(t) \ .$$

The inequality is assumed to hold almost surely, and '$*$' is the $(min, +)$ convolution operator, defined as $A * \mathcal{S}(t) = \inf_{0 \leq s \leq t}\{A(s) + \mathcal{S}(s,t)\}$.

This paper uses two types of statistical service curves for general and FIFO scheduling. General scheduling, or aggregate scheduling [6], does not specify any arbitration order among the flows. Suppose that an arrival flow $A(t)$, together with some cross arrival flow $A_c(t)$, are served at a node with capacity C and general scheduling. Then, the node offers the flow $A(t)$ the so called leftover service curve $\mathcal{S}(s,t)$ satisfying for all $0 \leq s \leq t$ [23]

$$\mathcal{S}(s,t) = \left[C(t-s) - A_c(s,t)\right]_+ \ , \tag{3}$$

where $[x]_+$ denotes the positive part $max(x, 0)$ of a real number x. Note that this service curve holds in the worst-case when $A(t)$ receives the lowest priority at a static priority scheduler, whence the name of a leftover service curve. In turn, if $A(t)$ and $A_c(t)$ are served according to a FIFO policy, then the node offers the flow $A(t)$ the service curve

$$\mathcal{S}(s,t) = \left[C(t-s) - A_c(s,t-x)\right]_+ 1_{\{t-s>x\}} \ , \tag{4}$$

where $x \geq 0$ is a free parameter, and $1_{\{\cdot\}}$ is the indicator function. The FIFO service curve recovers the leftover service curve from Eq. (3) by setting $x = 0$, and it provides tighter lower bounds on the service. FIFO service curve were previously proposed in the context when $A_c(t)$ has a deterministic envelope, i.e., a function $\mathcal{G}_c(t)$ satisfying $A_c(t) - A_c(t) \leq \mathcal{G}_c(t-s)$ for all $0 \leq s \leq t$ [16], [6]. In the probabilistic framework herein, Eq. (4) can be proven by extending the proof of Theorem 6.2.1 from [6] to sample-paths and bivariate processes.

Having a service curve definition with random processes it is convenient to assume the existence of MGF bounds [23]. Unlike bounding the arrivals from above as in Eq. (2), the service curves are bounded from below. Concretely, a statistical service curve $S(s,t)$ has an MGF bound if for some choice of $\theta > 0$ there exists a prefactor K and a rate $r > 0$, both depending on θ, such that for all $0 \leq s \leq t$

$$E\left[e^{-\theta S(s,t)}\right] \leq K e^{-\theta r(t-s)} \ . \tag{5}$$

At a single node, backlog and delay bounds for an arrival process $A(t)$ can be derived in terms of a sum containing the MGF bounds on the arrivals and service of $A(t)$ [23]. In turn, in a multi-node scenario, if $A(t)$ crosses H nodes in series, each offering a service curve $\mathcal{S}^h(s,t)$, $h = 1, \ldots, H$, then $A(t)$ is offered the (network) service curve [9]

$$\mathcal{S}(s,t) = \mathcal{S}^1 * \mathcal{S}^2 * \ldots * \mathcal{S}^H(s,t) \ . \tag{6}$$

With this service curve, single node results can be directly applied to derive end-to-end backlog and delay bounds for the arrivals of $A(t)$.

III. Buffer Scaling

In this section we address the problem of buffer dimensioning in the network from Figure 2 such that the loss probabilities at the two nodes are below a certain threshold. The network has two nodes traversed by n MMOO *through* flows $A_i(t)$ whose aggregate is denoted by $A^1(t)$. Each node is also traversed by $N - n$ *cross* flows whose aggregates are denoted by $A_1(t)$ and $A_2(t)$, respectively. Both nodes have the scaled capacity $C = Nc$ with the per-flow normalized

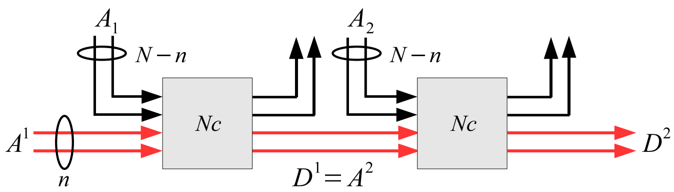

Fig. 2: Two network nodes with normalized capacities $C = Nc$ traversed by n through flows and $N-n$ cross flows.

capacity $c > 0$. For simplicity of notation we assume that all flows are homogeneous.

We provide two theorems to compute the buffer overflow probabilities at both nodes from Figure 2. The first one uses a direct analysis, while the second one provides a justification to derive the buffer overflow probability at the second node using a decomposition approach. The results from the theorems are illustrated with numerical examples.

Theorem 1: (TOTAL BACKLOG BOUNDS) Consider the network scenario from Figure 2 described above. All flows are statistically independent and are served according to FIFO scheduling. Each MMOO flow is bounded by an MGF envelope with rate r depending on $\theta > 0$ such that $r < c$. Denote by $\phi = \frac{n}{N}$ the proportion of through flows and by $\rho = \frac{r}{c}$ the node utilization, relative to θ. Then we have the following probabilistic bounds on the two backlogs.

1) FIRST NODE: The backlog process $B^1(t)$ at the first node satisfies for all $t, \sigma \geq 0$
$$Pr\Big(B^1(t) > \sigma\Big) \leq \inf_{\theta > 0} \left\{ \frac{e}{1-\rho} e^{-\theta\sigma} \right\} . \qquad (7)$$

2) SECOND NODE: The backlog process $B^2(t)$ at the second node satisfies for all $t, \sigma \geq 0$
$$Pr\Big(B^2(t) > \sigma\Big) \leq \inf_{\theta > 0} \left\{ \frac{Ke}{1-\rho} e^{-\theta\sigma} \right\} , \qquad (8)$$
where $K = \left(\frac{e}{1-\rho}\frac{1-\phi\rho}{\phi\rho}\right)^{\phi\rho} \frac{1}{1-\phi\rho}$.

The main result of the theorem is the second backlog bound (the first backlog bound appears also in [13] and is restated here for completeness). The key idea to derive Eq. (8) is to represent the service received by the through traffic at the first node using the FIFO statistical service curve from Eq. (4). We note that for the same network scenario with FIFO scheduling another backlog bound at the second node was recently derived in [13], but under additional statistical independence assumptions between the interarrivals of the through flows and past backlog processes.

In the theorem, the first backlog bound grows as $O(1)$ in the total number of flows N. On the other hand, for any fixed number of through flows n, the factor K is uniformly bounded in N which implies that the backlog bound at the second node also grows as $O(1)$. It then follows inductively that for a network with cross traffic and more than two nodes the backlog bounds at the downstream nodes are also invariant to the number of flows N.

The $O(1)$ order of growth immediately extends to the problem of buffer sizing in order to guarantee some loss probability. This follows from bounding the cell loss probability in a finite buffer system with the buffer overflow probability in an infinite buffer system, up to some constant invariant to N (see [36], [34]).

PROOF. Fix $t \geq 0$, $\theta > 0$ such that $r < c$, and σ. We first give the proof to derive the first backlog bound and later use it to derive the second backlog bound.

Denote the total arrival process at the first node by $A(t) = A^1(t) + A_1(t)$, and the corresponding departures by $D(t)$. Because the node offers the arrivals $A(t)$ a service curve $\mathcal{S}(s,t) = Nc(t-s)$ [9], we can write for the first backlog
$$\begin{aligned} B^1(t) &= A(t) - D(t) \\ &\leq A(t) - A * \mathcal{S}(t) \\ &\leq \sup_{0 \leq s \leq t} \{A(s,t) - Nc(t-s)\} \ . \end{aligned}$$

For $0 \leq s \leq t$, let a discretization parameter τ_0, and denote $j = \lfloor \frac{t-s}{\tau_0} \rfloor$ the integer part of $\frac{t-s}{\tau_0}$. Then we can write
$$\begin{aligned} & Pr\left(B^1(t) > \sigma\right) \\ & \leq Pr\left(\sup_{0 \leq s \leq t} \{A(s,t) - Nc(t-s)\} > \sigma \right) \\ & \leq Pr\left(\sup_{j \geq 1} \{A(t - j\tau_0, t) - Nc(j-1)\tau_0\} > \sigma \right) \\ & \leq \sum_{j \geq 1} e^{\theta N r j \tau_0} e^{-\theta N c j \tau_0} e^{\theta N c \tau_0} e^{-\theta\sigma} \\ & \leq \frac{e^{\theta N c \tau_0}}{\theta N (c-r) \tau_0} e^{-\theta\sigma} \ . \end{aligned} \qquad (9)$$

In the third line we used the monotonicity of the arrival processes. In the fourth line we applied Boole's inequality and in the last line we used the inequality $\sum_{j\geq 1} e^{-aj} \leq \int_0^\infty e^{-ax} dx$, for all $a > 0$. The proof for the first backlog is complete after optimizing $\tau_0 = \frac{1}{\theta N c}$ and minimizing over θ.

To derive the second backlog bound let us fix $s \geq 0$ and a parameter $x > 0$. For $0 < u < s - x$ let a discretization parameter τ_0, and denote $j = \lfloor \frac{s-x-u}{\tau_0} \rfloor$ the integer part of $\frac{s-x-u}{\tau_0}$.

Recall that $\mathcal{S}^1(s,t) = [C(t-s) - A_1(s, t-x)]_+ 1_{\{t-s > x\}}$ is a statistical service curve given by the first node to the through flow $A^1(t)$ (see Eq. (4)). Then we have the following bounds on the MGF of the departure process $D^1(s,t)$ of the through flow at the first node
$$\begin{aligned} & E\left[e^{\theta D^1(s,t)}\right] \leq E\left[e^{\theta\left(A^1(t) - A^1 * \mathcal{S}^1(s)\right)}\right] \\ & \leq E\left[\sup_{0 \leq u \leq s} e^{\theta\left(A^1(u,t) - [C(s-u) - A_1(u, s-x)]_+ 1_{\{s-u > x\}}\right)} \right] \\ & \leq E\left[e^{\theta A^1(s-x,t)}\right] + E\Big[\sup_{j \geq 1} e^{\theta A^1(s-x-j\tau_0, t)} \\ & \qquad e^{-\theta(C(x+(j-1)\tau_0) - A_1(s-x-j\tau_0, s-x))} \Big] \end{aligned}$$

$$\leq e^{\theta nr(t-s+x)} \left(1 + e^{-\theta C(x-\tau_0)} \sum_{j\geq 1} e^{-\theta(Nc-Nr)j\cdot} \right.$$

$$\leq e^{\theta nr(t-s)} \left(e^{\theta nrx} + \frac{e}{1-\rho} e^{-\theta(Nc-nr)x} \right) .$$

In the third line we restricted the supremum to $0 \leq u < s$ and applied $sup(a,b) \leq a+b$ for positive a and b. Then computed the sums and optimized τ_0 as in the derivatio Eq. (9).

To continue the previous derivation we use the follov infimum

$$\inf_{x>0} \left\{ \alpha e^{-\beta x} + e^{\gamma x} \right\} = \left(\frac{\alpha\beta}{\gamma} \right)^{\frac{\gamma}{\beta+\gamma}} \frac{\beta+\gamma}{\beta} ,$$

obtained using convex optimizations. Using this result we

$$E\left[e^{\theta D^1(s,t)}\right] \leq e^{\theta nr(t-s)} \left(\frac{e}{1-\rho} \frac{Nc-nr}{nr} \right)^{\frac{nr}{Nc}} \frac{N}{Nc- \ldots}$$
$$= K e^{\theta nr(t-s)} .$$

Therefore a bound on the MGF of the departures $D^1(s,t)$ is given by the bound on the MGF of the arrivals $A^1(s,t)$ multiplied by the factor K. It then follows that by repeating the steps from Eq. (9) the backlog bound at the second node is the backlog bound at the first node multiplied by K which completes the proof. □

Figure 3 illustrates on a logarithmic scale the backlog bounds derived with Theorem 1 at the first and the second nodes from Figure 2 as functions of nodes' utilization. The parameters of the MMOO flows are from Table I. Two fractions of through flows, i.e., $\phi = 0.1$ and $\phi = 0.9$ are considered, and the violation probability is set to 10^{-3}. The figure shows that for very low fractions of through flows, the backlog bounds at the two nodes become almost indistinguishable. This indicates that FIFO schedulers do not change the statistical structure of a small number of flows at the output.

To further illustrate the role of the fraction $\phi = \frac{n}{N}$ when analyzing FIFO schedulers, let us closely analyze the buffer overflow probability from Eq. (8). Because $\lim_{\phi\to 0} \left(\frac{1}{\phi}\right)^{\phi} = 1$ in the term K while the remaining factor is uniformly bounded in ϕ, the second backlog bound converges to the first backlog bound when decreasing the fraction of through flows $\phi = \frac{n}{N}$. The next theorem provides a justification that this convergence holds under greater generality, i.e., when $N \to \infty$. Concretely, the next theorem shows that by increasing N the output of one flow converges to the input. This suggests a decomposition analysis [21], whereas the second node can be studied in isolation as if the first node was removed from the network.

Theorem 2: (INPUT-OUTPUT CONVERGENCE) Consider the first node from Figure 2. All flows are statistically independent and are served according to FIFO scheduling. Each MMOO flow is bounded by an MGF envelope with rate r depending on $\theta > 0$ such that $r < c$. Denote by $\rho = \frac{r}{c}$ the node utilization, relative to θ. Then we have the following

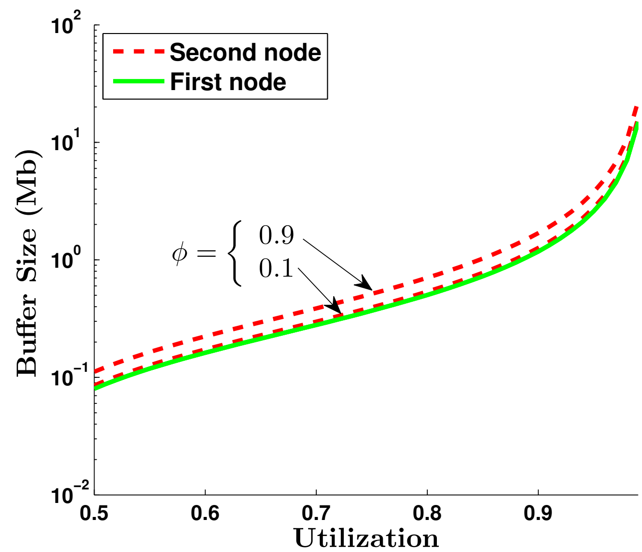

Fig. 3: Backlog bounds at the first and the second nodes as a function of utilization (per-flow average rate 25.6 $Kbps$ and peak rate $P = 64Kbps$, fraction of through flows $\phi = 0.1$ and $\phi = 0.9$, violation probability 10^{-3})

relationship between the input $A(t)$ and output $D(t)$ processes of a single flow:

$$Pr\left(D(t) \geq A\left(t - \frac{\log KN}{\theta c N}\right)\right) \geq 1 - \frac{1}{N} , \tag{10}$$

where $K = \frac{e}{1-\rho}$.

By letting $N \to \infty$ the theorem indicates that the output process $D(t)$ converges to the input process $A(t)$. This result is closely related to a result from [35] which states that, asymptotically, the effective bandwidths of flows remain unchanged at the output [35]. In the light of this known result, the contribution of the theorem is that it provides non-asymptotic bounds on the output, i.e., holding for any N, and further it provides a non-asymptotic justification that the per-flow effective bandwidths are preserved at the output for relatively small number of flows.

PROOF. Fix $t \geq 0$ and $\theta > 0$ such that $r < c$. The proof is based on deriving a bound on the delay process $W(t)$ of the single flow $A(t)$.

Let $d > 0$ and denote by $A_{N-1}(t)$ the rest of the arrival processes at the node. Recall now from Eq. (4) that for any $x > 0$ the process $\mathcal{S}(s,t) = [Nc(t-s) - A_{N-1}(s, t-x)]_+ 1_{\{t-s>x\}}$ is a statistical service for the flow $A(t)$. We can now bound the delay as follows

$$\begin{aligned} & Pr\left(W(t) > d\right) \\ & \quad \leq Pr\left(A(t-d) > D(t)\right) \\ & \quad \leq Pr\left(\sup_{0\leq s\leq t} \left(A(s,t-d) - \mathcal{S}(s,t)\right) > 0\right) \\ & \quad \leq Pr\left(\sup_{0\leq s< t-d} \left(A(s,t-d) - \mathcal{S}(s,t)\right) > 0\right) \end{aligned} \tag{11}$$

In the third line we used that $\mathcal{S}(s,t)$ is a service curv $A(t)$. In the last line we could restrict the supremum d the positivity of $\mathcal{S}(s,t)$.

We now choose $x = d$. For $0 < s < t-d$ let a discretiz parameter τ_0, and denote $j = \lfloor \frac{t-d-s}{\tau_0} \rfloor$ the integer pa $\frac{t-d-s}{\tau_0}$. Then we can continue Eq. (11) as follows:

$$
\begin{aligned}
Pr\left(W(t) > d\right) &\leq Pr\Bigg(\sup_{j \geq 1} \Big\{ A(t-d-(j+1)\tau_0, t \\
&\quad + A_{N-1}(t-d-(j+1)\tau_0, t-d) - Nc(d+j\tau_0) \\
&\leq e^{-\theta N c d} \sum_{j \geq 1} e^{-\theta N (c-r) j \tau_0} e^{\theta N c \tau_0} \\
&\leq K e^{-\theta N c d} \, .
\end{aligned}
$$

In the last line we optimized τ_0 as in Eq. (9). By equatin last expression above with $\frac{1}{N}$ we get

$$d = \frac{\log KN}{\theta c N} \, .$$

The proof is complete by observing from the definition of the delay process $W(t)$ that

$$Pr\Big(D(t) \geq A(t-d)\Big) \leq Pr\Big(W(t) \leq d\Big) \, .$$

$\square$

Figure 4 illustrates the offset $\frac{\log KN}{\theta c N}$ from Theorem 2 between the input and output processes of a single flow as a function of the total number of flows and for three utilization levels, i.e., $\rho = 0.5$, $\rho = 0.75$, and $\rho = 0.95$. The parameters of the MMOO flows are from Table I. Remarkably, for $N = 10^4$ flows and node utilization $\rho = 0.75$, the input-output relationship

$$D(t) \geq A(t - \tau)$$

holds for all times $t \geq 0$, with a violation probability $\varepsilon \approx 10^{-3}$ and $\tau \approx 1 \; ms$.

IV. Capacity Scaling

In this section we address the problem of capacity dimensioning for a network with voice traffic under specific constraints on the end-to-end delays experienced by each flow. First we derive explicit bounds on end-to-end delays, and then we numerically illustrate the influence of the number of flows on the required per-flow capacity and the achievable network utilization.

We consider the network scenario from Figure 5. The network has H nodes which are crossed by a through aggregate of n MMOO flows whose arrivals and departures are denoted by the processes $A(t)$ and $D(t)$, respectively. Moreover, each node is crossed by $N-n$ cross flows. The normalized capacity of each node is $C = Nc$. The assumption of FIFO scheduling from the previous section is now relaxed to general scheduling.

The next theorem provides end-to-end delay bounds for the flows of $A(t)$. As in the previous section we restrict the result to the simplified case of homogeneous flows.

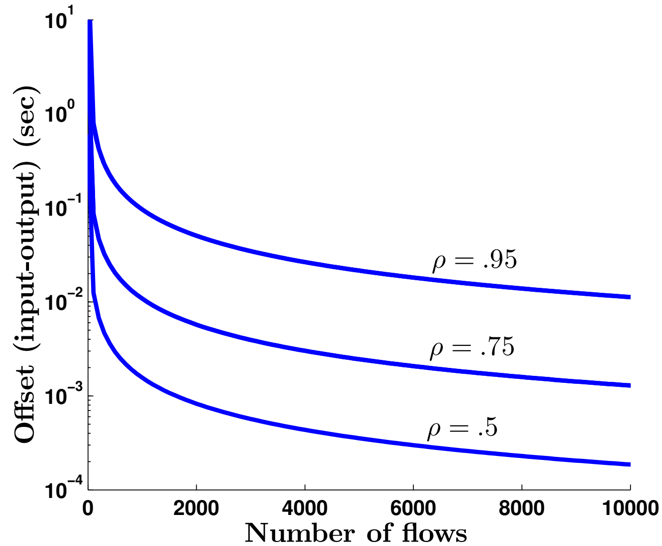

Fig. 4: Offsets between the input and output processes of a single flow as a function of the number of flows N (per-flow average rate 25.6 $Kbps$ and peak rate $P = 64Kbps$, and node utilizations $\rho = 0.5$, $\rho = 0.75$, and $\rho = 0.95$)

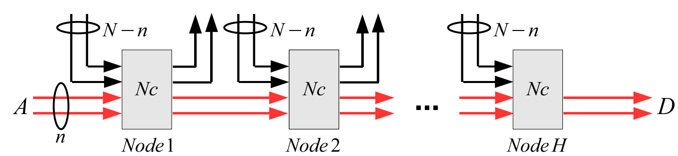

Fig. 5: A network with H nodes traversed by n through flows and $N-n$ cross flows; the normalized capacity at each node is $C = Nc$.

Theorem 3: (End-to-end Delay Bounds) Consider the network scenario from Figure 5 described above. We assume that each MMOO flow is bounded by an MGF envelope with rate r which depends on $\theta > 0$ such that $r < c$. All flows are statistically independent and are served according to general scheduling. Denote by $\phi = \frac{n}{N}$ the proportion of through flows and by $\rho = \frac{r}{c}$ the node utilization, relative to θ. Then the end-to-end delay $W(t)$ satisfies for all $t, d \geq 0$

$$Pr\Big(W(t) > d\Big) \leq \inf_{\theta > 0} \left\{ K e^{-\theta N c (1-(1-\phi)\rho) d} \right\} \, , \tag{12}$$

where $K = \left(\frac{e\left(\frac{1}{H} + (1-\phi)\rho\right)}{1-\rho} \right)^H$.

In the theorem, the end-to-end delay bound decreases exponentially fast in the number of flows N, or in the capacity $C = Nc$. By fixing an end-to-end delay bound d on the through flows, the required per-flow capacity to meet this constraint with some violation probability ε is given by the implicit equation

$$c \leq \frac{1}{N} \inf_{\{\theta \,:\, \theta > 0, \; c > r\}} \left\{ \frac{1}{\theta\left(1-(1-\phi)\rho\right) d} \log \frac{K}{\varepsilon} \right\} \, . \tag{13}$$

Using $\rho < 1$ in the right-hand side yields $c = O\left(\frac{1}{N}\right)$.

PROOF. Recall from Eq. (3) that the processes $\mathcal{S}^h(s,t$ $[C(t-s) - A_h(s,t)]_+$ are statistical service curves for through aggregate at each node h. Moreover, the process

$$\mathcal{S}(s,t) = \mathcal{S}^1 * \mathcal{S}^2 * \ldots * \mathcal{S}^H(s,t)$$

is a statistical network service curve for $A(t)$ across th nodes (see Eq. (6)).

Let us compute now an MGF bound for $\mathcal{S}(s,t)$ for s $0 \leq s \leq t$. Denoting $x_0 = s$ and $x_H = t$ we can write

$$E\left[e^{-\theta\mathcal{S}(s,t)}\right] \leq E\left[\sup_{x_0 \leq \cdots \leq x_H} e^{-\theta\sum_h \mathcal{S}^h(x_{h-1},x_h)}\right]$$
$$\leq e^{-\theta C(t-s)} E\left[\sup_{x_0 \leq \cdots \leq x_H} e^{\theta\sum_h A_h(x_{h-1},x_h)}\right]$$

For a discretization parameter τ_0 denote by $j_h = \lfloor \frac{t-}{}$ the integer parts of $\frac{t-x_j}{\tau_0}$ for all $j = 1, \ldots, H-1$. We now bound the last expectation with

$$\sum_{0 \leq j_1 \leq \cdots \leq j_{H-1} \leq \lfloor \frac{t-s}{\tau_0} \rfloor} E\left[e^{\theta(A_1(s,t-j_1\tau_0) + \cdots + A_H(t-(j_{H-1}+1)\tau_0,t))}\right]$$

Using the statistical independence of the processes $A_h(s,t)$ and the bounds on their MGFs we obtain

$$\begin{aligned}
&E\left[e^{-\theta\mathcal{S}(s,t)}\right] \\
&\leq e^{(H-1)\theta r_1\tau_0} e^{-\theta(C-r_1)(t-s)} \sum_{0 \leq j_1 \leq \cdots \leq j_{h-1} \leq \lfloor \frac{t-s}{\tau_0} \rfloor} 1 \\
&\leq \binom{\lfloor \frac{t-s}{\tau_0} \rfloor + H - 1}{H-1} e^{(H-1)\theta r_1\tau_0} e^{-\theta(C-r_1)(t-s)} ,
\end{aligned}$$

where $r_1 = (N-n)r$. In the last line the binomial coefficient is the number of combinations with repetitions.

Similarly as in Eq. (11) we have that

$$Pr\left(W(t) > d\right) \leq Pr\left(\sup_{0 \leq s < t-d} \left(A(s,t-d) - \mathcal{S}(s,t)\right) > 0\right) .$$

To further derive a sample path bound we discretize as in the proof of Theorem 1 and use the bound on the MGF of $\mathcal{S}(s,t)$ derived above. It follows that

$$\begin{aligned}
&Pr\left(W(t) > d\right) \\
&\leq e^{-\theta(C-r_1)d} e^{\theta C\tau_0} e^{\theta H r_1\tau_0} \sum_{j \geq 1} \binom{j+H-1}{H-1} e^{-\theta N(c-r)j\tau_0} \\
&\leq \left(\frac{e^{\frac{\theta(C+Hr_1)\tau_0}{H}}}{\theta N(c-r)\tau_0}\right)^H e^{-\theta(C-r_1)d} .
\end{aligned}$$

In the last line we used $\sum_{j \geq 0} \binom{j+H-1}{H-1} a^j = \left(\frac{1}{1-a}\right)^H$ for all $0 < a < 1$ (see [23]) and the inequality $\left(\frac{1}{1-e^{-a}}\right)^H - 1 \leq \left(\frac{1}{a}\right)^H$ for all $a > 0$.

The proof is complete after optimizing $\tau_0 = \frac{H}{\theta(C+Hr_1)}$ and minimizing over θ. □

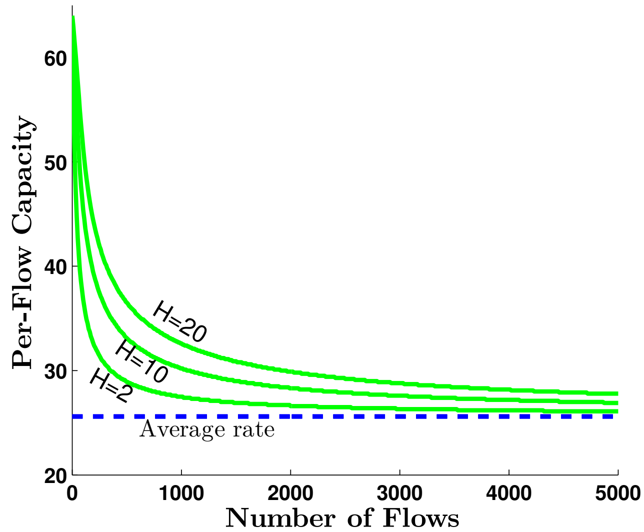

Fig. 6: Per-flow capacity (Mbps) as a function of the number of flows (per-flow average rate 25.6 $Kbps$ and peak rate $P = 64Kbps$, number of nodes $H = 2, 10, 20$, fraction of through flows $\phi = 0.5$, end-to-end delay 200 ms, violation probability $\varepsilon = 10^{-3}$)

Figure 6 illustrates the $O\left(\frac{1}{H}\right)$ decrease of the required per-flow capacity from Eq. (13) in order to guarantee an end-to-end delay of 200 ms for the through flows with a violation probability of 10^{-3}. The parameters of the MMOO flows are from Table I and the fraction of through flows is $\phi = \frac{n}{N} = 0.5$. The asymptotic decrease is depicted for various values $H = 2$, $H = 10$, and $H = 20$ of the number of nodes.

For the same numerical settings as above, Figure 7 illustrates the achievable node utilization as a function of capacity. The number of nodes is now set to $H = 10$ and various values of the fractions of through flows are considered, i.e., $\phi = 0.25$, $\phi = 0.5$, and $\phi = 0.75$. The figure shows that utilizations higher than 90% can be achieved at 100 $Mbps$, when the through flows dominate the cross flows (i.e. $\phi > 0.5$).

Both Figures 6 and 7 illustrate that Theorem 3 achieves significant statistical multiplexing gain. In fact, in regimes with capacities bigger than 100 $Mbps$, most of the statistical multiplexing gain is attained which suggests that the end-to-end delay bounds from Theorem 3 are tight despite the assumption of general scheduling at the nodes and the multiple application of Inequality 1. In other words, statistical multiplexing dominates the effects of link scheduling at high capacities. A similar observation has been reported in [27] in the case of a single node.

V. CONCLUSIONS

In this paper we have derived non-asymptotic backlog and end-to-end delay bounds in a network carrying voice traffic. By adopting a probabilistic network calculus approach and by using a probabilistic service curve available for FIFO scheduling, we have shown that different network nodes have similar

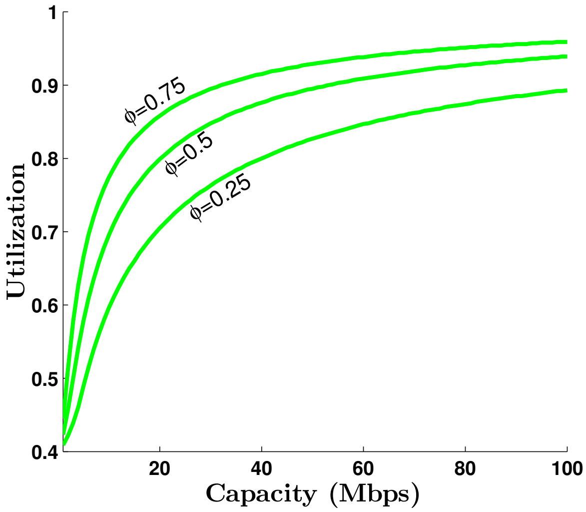

Fig. 7: Achievable node utilization as a function of capacity (per-flow average rate 25.6 $Kbps$ and peak rate $P = 64Kbps$, number of nodes $H = 10$, end-to-end delay 200 ms, violation probability $\varepsilon = 10^{-3}$)

buffer requirements in order to guarantee desirable loss probabilities. Remarkably, the buffers can be sized independently of the number of voice flows. Also, we have shown that in a network with general scheduling the end-to-end delays decay exponentially fast in the number of voice flows. Consequently, by fixing some end-to-end delay constraints, the required per-flow capacities decrease accordingly and the network can reach very high utilizations. The results from this paper hold in principle for traffic described by hyperexponential distributions which provide tighter descriptions for aggregates of MMOO flows and also cover broader classes of traffic.

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